

On the metallicity distribution in the nuclei of elliptical galaxies

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ABSTRACT

Using current models of spectrophotometric properties of single age, single metallicity stellar populations I have computed the Mg_2 , $H\beta$, $Fe52$ and $Fe53$ line strengths for stellar populations with a metallicity spread. The comparison of these models with the nuclear indices of early type galaxies yield the following major conclusions. The metallicity distribution of the closed box, simple model for the chemical evolution of galaxies is not able to account for Mg_2 and $Fe52$, $Fe53$ values in excess of $\sim 0.27, 3$ and 2.7 , respectively, which are observed in the nuclei of a large fraction of Ellipticals. To reproduce the line strengths in these galaxies high average metallicities, small metallicity dispersion and old ages are required. In particular, Mg_2 values of ~ 0.3 are reproduced only with a metallicity distribution ranging from $\sim 0.5Z_\odot$ to $\sim 3Z_\odot$, and ~ 15 Gyr old stellar populations. I interpret the data as indicating that the gas out of which the nuclei of ellipticals formed was pre-enriched, to larger metallicities for increasing Mg_2 . The presence of a metallicity dispersion does not alter the relation between Mg_2 and Iron indices with respect to the SSP models. Thus, the need for a Mg/Fe overabundance in the strongest lined galaxies is confirmed, and I present a simple way to estimate the $[Mg/Fe]$ ratio on the basis of existing models with solar abundance ratios.

Key words: Galaxies: elliptical & lenticular, stellar content, metallicity, formation

1 INTRODUCTION

The question of the age of the bulk of the stars in elliptical galaxies is still subject to a large debate. Opposed to the classical picture in which Ellipticals are basically inhabited by ~ 15 Gyr old stellar populations (see e. g. Renzini 1986), O’Connell (1986), amongst others, proposed that a substantial component of stars as young as 5 Gyr has to be present to account for the observed spectral energy distribution of Ellipticals. In more recent years many observational evidences have been found to support the notion of elliptical galaxies formation at high redshift, including the tightness of the colour–central velocity dispersion (σ) relation found for Ellipticals in Virgo and Coma by Bower, Lucey & Ellis (1992); the thinness of the fundamental plane (Renzini & Ciotti 1993) for the Ellipticals in the same two clusters; the modest passive evolution measured for cluster Ellipticals at intermediate redshifts (Franx & van Dokkum 1996, Bender, Ziegler & Bruzual 1996); the negligible luminosity evolution observed for the red galaxies (Lilly et al. 1995) and for early type galaxies in clusters in the redshift range $z < 1$ (Dickinson 1996); the detection of bright red galaxies at redshifts as large as 1.2 (Dickinson 1996). On the other hand, hints for a continuous formation of Ellipticals in a wide redshift range

have also been found: the relatively large $H\beta$ values measured in a sample of nearby Ellipticals, which could indicate a prolonged star formation activity in these galaxies, up to ~ 2 Gyr ago (Gonzalez 1993, Faber et al. 1995); the apparent paucity of high luminosity Ellipticals at $z \approx 1$ compared to now (Kauffmann, Charlot & White 1996).

Two competing scenarios have been proposed also for the process leading to the formation of the bulk of the stars in Ellipticals: early merging of lumps containing gas and stars (e.g. Bender, Burstein & Faber 1993), in which some dissipation plays a role in establishing the chemical structure of the outcoming galaxy; and the merging of early formed stellar systems, occurring in a wide redshift range, and preferentially at late epochs, following the hierarchical formation of structures (Kauffmann, White & Guiderdoni 1993).

In order to help understanding when and how elliptical galaxies formed I have computed synthetic spectral indices for stellar populations with a metallicity spread, and compared them to the corresponding observations in the nuclei of Ellipticals. The study of line strengths in the spectra of early type galaxies has shown to be a powerful tool for investigating the age and the metallicity of these systems (Faber et al. 1995; Fisher, Franx and Illingworth 1995; Buzzoni 1995b and references therein). With few exceptions

(Vazdekis et al. 1996, Bressan, Chiosi & Tantaló 1996), most of the authors have interpreted the observed line strengths through comparisons with theoretical models constructed for single age, single metallicity stellar populations (SSPs). The major results of these studies can be summarized as follows:

- i) the Mg_2 indices measured in elliptical galaxies are consistent with the notion that these systems are inhabited by old stellar populations, the differences in Mg_2 tracing differences in average metallicity (Buzzoni, Gariboldi & Mantegazza 1992). However, the difficulty in determining separately age and metallicity (Renzini 1986) weakens considerably this simple picture (Worthey 1994);
- ii) the $\text{H}\beta$ line strength offers an opportunity to break the age–metallicity degeneracy, if this index is measuring the temperature of turn-off stars (see Faber et al. 1995). In this view, the data derived for a sample of nearby Elliptical galaxies indicate that the ages of the stellar populations in their nuclei span a wide range, the weakest Mg_2 galaxies being the youngest (Gonzalez 1993, Faber et al. 1995);
- iii) the Magnesium to Iron abundance ratio in the nuclei of the highest Mg_2 Ellipticals is likely to be larger than solar (Gorgas, Efstathiou & Aragón Salamanca 1990; Worthey, Faber & Gonzalez (1992), hereinafter WFG; Davies, Sadler & Peletier 1993). Weiss, Peletier and Matteucci (1995) estimate $[\text{Mg}/\text{Fe}]$ ranging from 0.3 to 0.7 dex within the brightest ellipticals.

Real galaxies host composite stellar populations, with a spread in the major parameters like age and metallicity. This may apply also to the nuclei of galaxies, where typically $\sim 10^7 L_\odot$ are sampled, corresponding to ~ 100 bright globular clusters. The existence of a substantial metallicity spread in elliptical galaxies and bulges is supported by direct observations. For example, the Colour-Magnitude diagram of a field in M32 (Freedman 1989) shows a wide red giant branch, corresponding to stars spanning a metallicity range of 0.6 dex approximately. The K-giants in the galactic bulge have metallicities ranging from $\sim 0.1 Z_\odot$ to $\sim 5 Z_\odot$ (Rich 1988). Finally, the mere evidence for abundance gradients in elliptical galaxies, as inferred from line strengths gradients, indicates that a metallicity spread is present in these systems. This last argument applies to galaxies as a whole: whether or not their nuclei host stellar populations with a spread in metal content depends on the modalities of the galaxy formation. However, due to projection effects, a substantial fraction of the light measured in the nuclei of Ellipticals comes from regions located outside the three dimensional core. This fraction can e.g. amount to $\sim 50\%$ for King models (Binney & Tremaine 1987). Therefore a pure radial metallicity gradient translates into a metallicity spread in the stellar population contributing to the light measured in the galactic nuclei.

In this paper, the effect of a metallicity spread on the integrated indices is investigated, viewing a given galaxy (or a portion of it) as the sum of SSPs. These models are compared to the relevant observations to derive conclusions on the stellar content of the nuclear regions of early type galaxies and inferences on their formation process. Section 2 describes how the models are computed, and the results are presented in Section 3. In Section 4 the models are compared to the observational data, and in Section 5 the implications for the formation of elliptical galaxies are discussed. The main conclusions are summarized in Section 6.

2 COMPUTATIONAL PROCEDURE

The spectral indices considered in the present work are defined as follows (Burstein et al. 1984):

$$\text{Mg}_2 = -2.5 \text{ Log } \frac{F_1(\text{Mg}_2)}{F_c(\text{Mg}_2)} \quad (1)$$

$$\text{H}\beta = \Delta_\beta \times \left[1 - \frac{F_1(\text{H}\beta)}{F_c(\text{H}\beta)} \right] \quad (2)$$

$$\text{Fe52} = \Delta_{\text{Fe}} \times \left[1 - \frac{F_1(\text{Fe52})}{F_c(\text{Fe52})} \right] \quad (3)$$

$$\text{Fe53} = \Delta_{\text{Fe}} \times \left[1 - \frac{F_1(\text{Fe53})}{F_c(\text{Fe53})} \right] \quad (4)$$

where the various F_1 denote the fluxes measured within the spectral windows of the different lines, centered at $\lambda \simeq 5175, 4863, 5267$ and 5334 \AA for the Mg_2 , $\text{H}\beta$, Fe52 and Fe53 features, respectively. F_c are the pseudocontinuum fluxes measured at the line location, as interpolated from the fluxes measured in adjacent windows, and Δ_β , Δ_{Fe} are the wavelength widths of the windows in which the $\text{H}\beta$ and Iron indices are measured.

The above definitions apply to the spectra of single stars, of SSPs and of collections of SSPs, inserting the appropriate values for the fluxes. Therefore, for a collection of N simple stellar populations, each contributing a fraction Φ_S to the total bolometric flux F_{bol} of the composite stellar population, the integrated indices are given by equations (1) to (4) with

$$F_1 = F_{\text{bol}} \times \sum_{S=1}^N \left(\frac{F_1}{F_{\text{bol}}} \right)_S \Phi_S \quad (5)$$

$$F_c = F_{\text{bol}} \times \sum_{S=1}^N \left(\frac{F_c}{F_{\text{bol}}} \right)_S \Phi_S \quad (6)$$

where the subscript S refers to the single SSP. The spectral energy distribution of an SSP, and particularly the various flux ratios, are controlled by a number of parameters, including the metallicity (Z), age (t), helium content (Y) and the elemental abundances of the population. Thus, the integrated indices of a collection of SSPs depend on how the fractionary bolometric flux Φ_S is distributed over the range covered by all the relevant parameters. The problem of deriving these parameters from the observed line strengths can be simplified considering various suitable indices, each controlled by different parameters. For example, Gonzalez (1993) uses a combination of Magnesium and Iron line strengths (mostly sensitive to Z) and the $\text{H}\beta$ index (mostly sensitive to t) to determine age and metallicity of a sample of early type galaxies. Still, in order to map the integrated indices of collections of SSPs into their fundamental properties one needs to account for the presence of a possible spread in the parameters which control the various line strengths.

One possible approach to this problem consists in computing models for the chemical evolution of galaxies, which automatically yield the distribution of the SSPs over the fundamental parameters (e.g. Vazdekis et al. 1996, Tantaló et al. 1996). The output of these models, though, depends on the specific ingredients used, like the adopted star formation rate, initial mass function, ratio of dark to luminous matter, nucleosynthesis, criteria for the establishment of a

galactic wind, etc. A different approach consists in exploring the dependence of the various indices on the presence of a spread in the populations parameters adopting a physically motivated function Φ_S , and using relations (1) to (4). This approach, which has the advantage of allowing an easy exploration of the parameter space by simply changing the Φ_S functions, will be adopted here. Also, I will restrict to considering collections of SSPs all with the same age, but a substantial spread in metallicity. The results are then meant to describe the effects of the presence of a metallicity spread in a stellar population formed within a short time scale, so that the integrated indices are not appreciably influenced by age differences in the individual components.

2.1 Line strengths for simple stellar populations

In order to compute the integrated indices of composite stellar populations one has to know the F_1/F_{bol} and F_c/F_{bol} ratios of the SSPs as functions of Z and t . Available SSP models in the literature tabulate bolometric corrections, colours and spectral indices for different ages and metallicities. I then write:

$$\frac{F_1}{F_{\text{bol}}} = \left(\frac{F_1}{F_c}\right) \times \left(\frac{F_c}{F_{\text{bol}}}\right) \quad (7)$$

for each SSP, and approximate F_c with the flux in the V band, for Mg₂, Fe52 and Fe53, and with the B band flux for H β . Although the pseudocontinuum fluxes do not correspond precisely to the flux in the V or the B band, the main conclusions of this paper are not affected by this approximation. For example, the integrated Mg₂ indices computed with $F_c = F_B$ differ from those computed with $F_c = F_V$ by less than 2 percent.

Two sets of SSP models are used here: Buzzoni's models (Buzzoni et al. 1992; Buzzoni, Mantegazza & Gariboldi, 1994; Buzzoni 1995a), and Worthey's models (Worthey 1994), hereinafter referred to as B and W respectively. The metallicity range covered by W models goes from $0.01Z_{\odot}$ to $\sim 3Z_{\odot}$ ($Z_{\odot} \simeq 0.017$), while B models encompass a larger Z range, from $\sim 0.006Z_{\odot}$ to $\sim 6Z_{\odot}$. Each metallicity is characterized by one value for the helium abundance: Worthey (1994) assumes $Y = 0.228 + 2.7Z$, while in B models Y increases less steeply with Z , according to $Y \simeq 0.23 + Z$. Besides, the isochrones used to construct the SSPs have solar abundance ratios. This corresponds to specifying the chemical trajectory followed in the evolution of the composite stellar population.

The two sets of models present systematic differences in the bolometric output, broad band colours and line strengths (Worthey 1994, Buzzoni 1995b). At least part of these differences can be ascribed to the different choices of the $\Delta Y/\Delta Z$ parameter (Renzini 1995) and to the different fitting functions (i.e. the dependence of individual stellar indices on effective temperature, gravity and metallicity) adopted in the computations.

2.2 The metallicity distribution functions

The contribution of a given SSP to the bolometric light of a composite stellar population of total mass M_T and total luminosity L_T can be written as

$$\Phi_S = \frac{L_S}{L_T} = \left(\frac{L}{M}\right)_S \frac{M_S}{M_T} \frac{M_T}{L_T}. \quad (8)$$

The distribution function Φ_S is then proportional to the mass distribution over the metallicity range, through the inverse of the $(M/L)_S$ ratio, which depends on the metallicity and helium content (Renzini 1995).

The closed box, simple model for the chemical evolution of galaxies predicts that the distribution of the stellar mass over the total metallicity follows the relation $f(Z) \propto e^{-Z/y}$ (Tinsley 1980), where y is the stellar yield. According to this model, most stars are formed at the lowest metallicities. More sophisticated models for the chemical evolution of elliptical galaxies, which take into account the occurrence of galactic winds (Arimoto & Yoshii, 1987; Matteucci & Tormambé, 1987), also predict the existence of a substantial metallicity spread in the stellar content of elliptical galaxies. In these models, the more massive the galaxy, the later the galactic wind sets in, and further chemical enrichment is achieved. Correspondingly, as the galactic mass increases, the metallicity distributions get skewed towards higher Z values. Nevertheless, a substantial fraction of low Z stars is always present, and indeed Arimoto & Yoshii's metallicity distributions (by number) for model Ellipticals with mass ranging from 4×10^9 to $10^{12} M_{\odot}$ are well described by a closed box model relation :

$$f(Z) = \frac{\exp(-Z/y)}{\int_{Z_m}^{Z_M} \exp(-Z/y) dZ} \quad (9)$$

with a minimum metallicity $Z_m \simeq 0.01Z_{\odot}$, a maximum metallicity Z_M increasing from $\sim 2Z_{\odot}$ to $\sim 6Z_{\odot}$, and y varying from $2Z_{\odot}$ to $3Z_{\odot}$. So, the metallicity range spanned by the stars in these model ellipticals goes from ~ 2.3 to ~ 2.8 dex.

Relation (9) finds also observational support from the direct determination of the metallicity distribution of K-giants in the bulge of our galaxy, which appear to follow a close box model relation with $Z_M \sim 5Z_{\odot}$ and $y \simeq 2Z_{\odot}$ (Rich 1988). McWilliam and Rich (1994) revised Rich (1988) metallicity scale for [Fe/H] towards values lower by ~ 0.3 dex, but find a [Mg/Fe] overabundance of the same amount, and state that the distribution of [(Fe+Mg)/H] in the bulge stars may agree with Rich (1988) [Fe/H] distribution.

I then adopt eq. (9) to describing the M_S/M_T distribution over the metallicity for the composite stellar populations, and explore the effect of different values for the three parameters Z_M , Z_m and y . For a fixed (low) Z_m , increasing values of Z_M (and yield) are meant to describe stellar populations produced in an environment in which the chemical processing is terminated at progressively higher levels of completion. These sequences of models, then, conform to the predictions of chemical evolution models with galactic winds for increasing galactic mass. Different values of Z_m , instead, characterize different degrees of pre-enrichment of the gas.

To derive the Φ_S distribution the behaviour of the (M/L) ratio for the SSPs with increasing metallicity and helium content needs to be specified. In B models, which are characterized by a low $\Delta Y/\Delta Z$ parameter, M/L increases with Z , going from 2.2 to 4.5 for Z ranging from $0.01Z_{\odot}$ to $3Z_{\odot}$, for the 15 Gyr old SSPs. In W 17 Gyr old models, the mass to (bolometric) luminosity ratio increases mildly

with metallicity up to a maximum value of 4.2 reached at $0.5Z_{\odot}$, and it decreases afterwards, down to 3.3 at $3Z_{\odot}$. While the different behaviour reflects the different $\Delta Y/\Delta Z$ (Renzini 1995), these M/L values are not directly comparable, the total mass M being computed with different prescriptions (Worthey 1994). Besides this, neither of the two M/L correspond to what should be inserted in eq. (9): Buzzoni's M values do not take into account the mass locked into stellar remnants; Worthey's M values do not include the remnant masses for stars born with $M > 2 M_{\odot}$, but do include both the remnant and the returned mass for stars born with mass between the turn-off mass and $2M_{\odot}$. I then chose to neglect the dependence of $(M/L)_S$ on the metallicity of the SSPs, and adopt $\Phi_S \propto f(Z)$. If M/L increases with metallicity, as in B models, this approximation leads to an overestimate of the contribution of the high metallicity SSPs in the integrated indices for the composite stellar populations. If, on the contrary, the M/L given in W models are more appropriate, the contribution of the high Z populations will be underestimated. Notice, however, that in W models M/L varies by only a factor of 1.2 for Z varying from $0.01Z_{\odot}$ to $3Z_{\odot}$. In the following, composite stellar populations with this Φ_S distribution function will be briefly referred to as CSPs.

3 RESULTS OF THE COMPUTATIONS

Following the prescriptions described in Sect. 2, the integrated spectral indices are given by the following relations:

$$\text{Mg}_2 = -2.5 \text{ Log } \frac{\int_{Z_m}^{Z_M} dZ \ 10^{-0.4 (Mg_2^S - BC_V^S)} e^{-Z/y}}{\int_{Z_m}^{Z_M} dZ \ 10^{0.4 BC_V^S} e^{-Z/y}} \quad (10)$$

$$\text{H}\beta = \frac{\int_{Z_m}^{Z_M} dZ \ \text{H}\beta^S \ 10^{0.4 BC_B^S} e^{-Z/y}}{\int_{Z_m}^{Z_M} dZ \ 10^{0.4 BC_B^S} e^{-Z/y}} \quad (11)$$

$$\text{Fe52} = \frac{\int_{Z_m}^{Z_M} dZ \ \text{Fe52}^S \ 10^{0.4 BC_V^S} e^{-Z/y}}{\int_{Z_m}^{Z_M} dZ \ 10^{0.4 BC_V^S} e^{-Z/y}} \quad (12)$$

$$\text{Fe53} = \frac{\int_{Z_m}^{Z_M} dZ \ \text{Fe53}^S \ 10^{0.4 BC_V^S} e^{-Z/y}}{\int_{Z_m}^{Z_M} dZ \ 10^{0.4 BC_V^S} e^{-Z/y}} \quad (13)$$

where BC_B and BC_V denote the bolometric corrections to the B and the V band magnitudes, respectively, and the S index denotes the SSP quantities, which depend on metallicity and age. One can immediately notice that, since high Z populations yield a relatively low contribution in the B and V bands, the low metallicity component of a composite stellar population will tend to dominate the integrated indices.

The behaviour of these indices as functions of the average metallicity of a composite stellar population has been explored by considering different values for Z_m and Z_M . In order to characterize each distribution in terms of one parameter, an *average metallicity* has been computed, defined by the following relation:

$$[\langle \text{Fe}/\text{H} \rangle] = \text{Log } \frac{\int_{Z_m}^{Z_M} dZ \ \frac{(Z/X)^S}{(Z/X)_{\odot}} e^{-Z/y}}{\int_{Z_m}^{Z_M} dZ \ e^{-Z/y}}. \quad (14)$$

Other definitions of *average metallicity* can be found in the literature (see e.g. Arimoto & Yoshii 1987). In this respect, it should be noticed that the quantity $\langle [\text{Fe}/\text{H}] \rangle$ differs systematically from $[\langle \text{Fe}/\text{H} \rangle]$ given by eq. (14) because the former assigns more weight to the low metallicity tail of the distribution. The difference between the two quantities amounts to ~ 0.2 dex for the widest Z distribution considered here ($0.01Z_{\odot} < Z < 6Z_{\odot}$). The choice of using eq. (14) is motivated by the fact that this parameter better describes the mass fraction of metals in the CSP, for the given $f(Z)$ distribution function.

3.1 Indices versus metallicity using B models

I will now describe the results of the integration of equations (10) to (13) for the different Z distributions, using B and W sets of SSP models. Among the various models by Buzzoni I have considered those computed with Salpeter IMF and red horizontal branches. Analogous assumptions characterize Worthey's models.

Figure 1 displays the integrated indices obtained using B SSPs at 15 Gyr, which are shown as thick lines in the four panels. The thin lines illustrate the effect of the presence of a metallicity spread (see captions), the lowest of which corresponds to $Z_m = 0.01Z_{\odot}$, and shows the expected behaviour of the indices for galaxies of increasing mass, in the frame of the wind models for the chemical evolution of galaxies. The different lines show the results obtained with different Z_m , describing the effect of assuming different degrees of pre-enrichment of the gas.

The loci described by the CSPs for increasing average metallicity are shallower than the pure SSP relations: this is due to the fact that the higher Z populations contribute less than the lower metallicity ones in the optical bands, where the considered spectral indices are measured. As a consequence, the Mg_2 and Iron indices of these CSPs never reach values as high as those characteristic of the highest Z SSPs, unless the metallicity spread is extremely small. Had I used the B-band flux as a measure of the pseudocontinuum for calculating the integrated Mg_2 index the effect would be stronger, since the high metallicity populations would receive even less weight. Taking into account the $(M/L)_S$ ratio dependence on Z as given in B models, when computing Φ_S , would also strengthen the difference between SSP and CSP models.

At any given average metallicity, the Mg_2 and Iron indices for CSPs are weaker (and $\text{H}\beta$ is stronger) than the corresponding values for SSPs. For example, a value of $[\langle \text{Fe}/\text{H} \rangle] = 0.22$ can be obtained with a composite stellar populations with parameters $(Z_m, Z_M, y) = (0.01, 5, 2) \times Z_{\odot}$. The differences between the line strengths of such a composite population and those of the SSP with the same $[\text{Fe}/\text{H}]$ amount to $\Delta \text{Mg}_2 \simeq -0.05$, $\Delta \text{H}\beta \simeq 0.18$, $\Delta \text{Fe52} \simeq -0.5$ and $\Delta \text{Fe53} \simeq -0.4$. These results refer to the $y = 2 \times Z_{\odot}$ case. Adopting $y = 4 \times Z_{\odot}$ (dotted lines), so as to enhance the fraction of the high Z component, the differences are only slightly smaller. This effect is particularly important when dealing with the highest metallicity galaxies.

In much the same way, **at any given value of the considered spectral index, the average metallicity of a composite stellar population is higher than the**

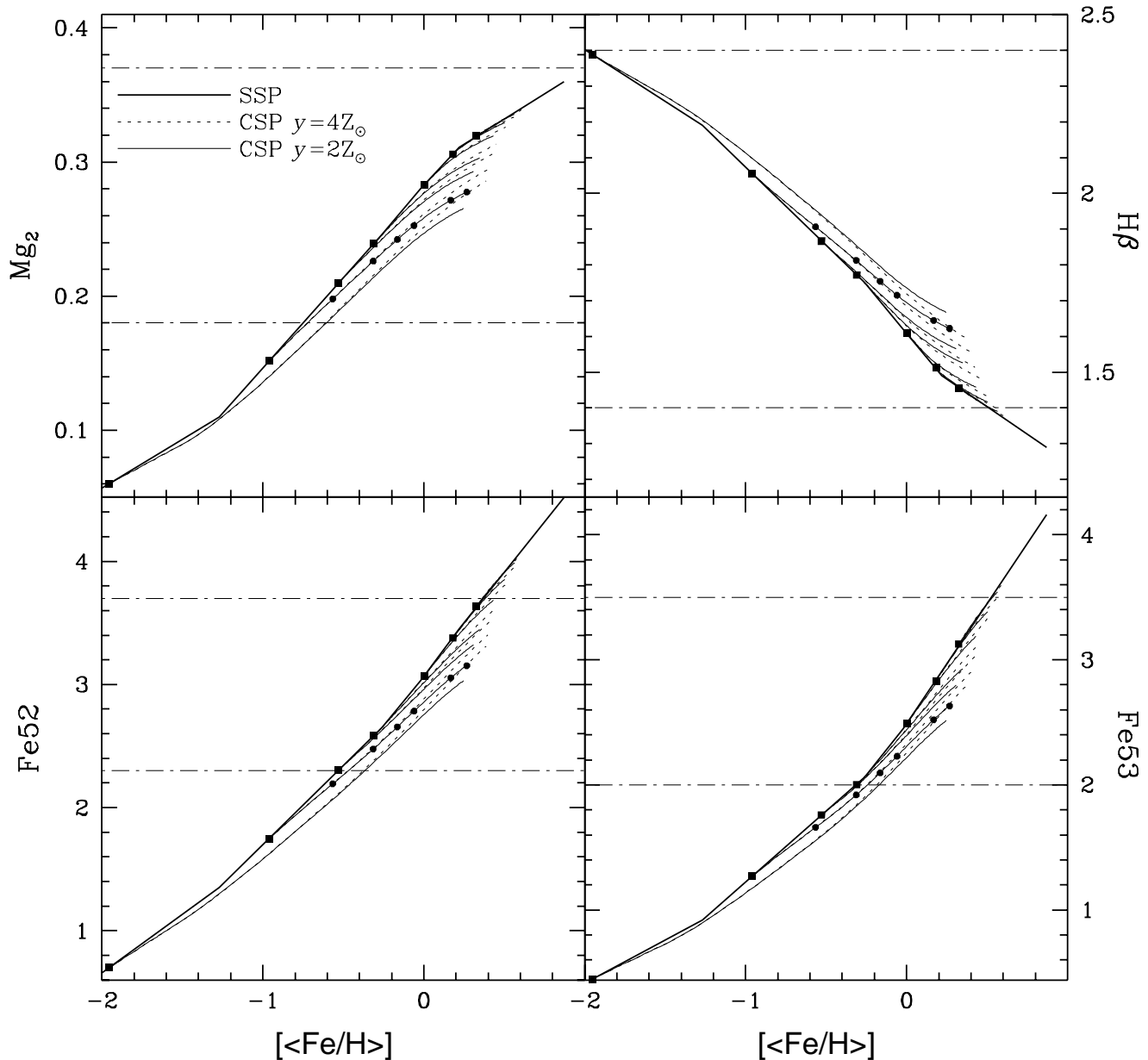


Figure 1. Model line strengths using Buzzoni's 15 Gyr old SSPs, as functions of metallicity. The thick lines are the loci of pure SSP models. The thin lines display models for CSPs for various values of the Z_m parameter. The following cases are shown: $Z_m/Z_\odot = (0.01, 0.1, 0.3, 0.5, 1, 1.5, 2)$, and the filled squares mark the values for SSPs with $Z = Z_m$. Along each line, Z_m varies from Z_m up to 0.1. The filled circles mark CSP models with $Z_m = 0.1Z_\odot$, and $Z_m = (0.5, 1, 1.5, 2, 4, 6) \times Z_\odot$. The effect of different values of the parameter y is also shown. Finally, the dot-dashed lines show the typical ranges spanned by observational data for the nuclei of elliptical galaxies.

metallicity of the SSP which has the same index. This is illustrated in Figure 2, where I plot, as a function of Mg_2 , the difference ($\Delta[Fe/H]$) between the average metallicity of the CSPs and that of SSP models with the same value of the Mg_2 index. Along any line, the metallicity distributions have the same Z_m and increasing Z_m , as in Figure 1. Figure 2 shows that the metallicity inferred from a given Mg_2 value using SSP models is lower than what would be derived using CSP models, the difference being larger the wider the metallicity distribution. The lower weight received by the high Z populations in eq. (10) causes the rapid growth of $\Delta[Fe/H]$

as the high Mg_2 ends of the curves are approached: high Mg_2 line strengths are obtained only with very large Z_m values. This is not a small effect: at $Mg_2 = 0.26$ the difference in the metallicities amounts to 0.3 dex, for a metallicity distribution extending down to $Z_m = 0.01Z_\odot$.

Adopting a large y the effect is still important, while it gets small only reducing the widths of the metallicity distributions, (i.e. increasing Z_m), as obvious. It follows that the calibration of Mg_2 in terms of metallicity via the comparison with theoretical values for SSPs is affected by a systematic

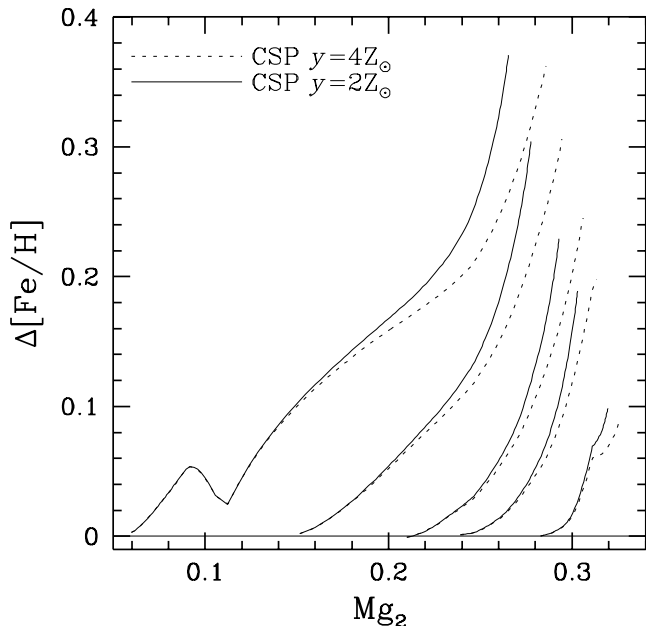


Figure 2. The difference between the average metallicity of CSPs and the metallicity of SSPs having the same Mg_2 index, as a function of the index itself, for Buzzoni's 15 Gyr old models. The different lines correspond to CSPs with different values for the Z_m parameter: $(0.01, 0.1, 0.3, 0.5, 1) \times Z_\odot$. As in Fig.1, along the lines Z_m increases up to $Z=0.1$. The minimum in the line corresponding to $Z_m = 0.01Z_\odot$ is an artifact of the linear interpolation between SSP models used in the computation. The effect of adopting different y values is shown.

effect, which can be considerably large, if a CSP spanning a substantial metallicity range is present.

The dot-dashed lines in Figure 1 show the typical range spanned by the observational data (e.g. Davies et al. 1987; WFG; Carollo, Danziger & Buson 1993). It appears that galaxies with $(Mg_2, Fe52, Fe53)$ up to $\sim (0.26, 3., 2.6)$ can be interpreted as hosting composite stellar populations with a metallicity distribution as in the closed box model. For these galaxies, higher metallic line strengths correspond to larger values for Z_m , and because of the $Mg-\sigma$ relation (Bender, Burstein & Faber 1993), to more massive objects. It should be noticed, however, that it is not possible to constrain the metallicity distribution from the integrated indices: the same value can be obtained with different metallicity ranges for the CSP, not for saying different distribution functions. Nevertheless, it seems that the observed metallic line strengths in the nuclei of the less luminous Ellipticals are consistent with the theoretical expectations from models for the chemical evolution which include the occurrence of galactic winds.

On the contrary, galaxy centers with metallic indices in excess than the above quoted values are not reproduced by these models (see also Casuso et al. 1996, Vazdekis et al. 1996), and appear instead to require some degree of pre-enrichment. The highest Mg_2 are barely accounted for by pure SSP models, and the strongest Iron indices are not reproduced by the CSP model with $0.1Z_\odot \leq Z \leq 6Z_\odot$, in which the fraction of stars with sub-solar metallicity is less

than 0.4. As can be seen in Figure 1, adopting a yield as high as $4Z_\odot$ does not alter these conclusions.

The $H\beta$ values measured in the highest metallicity Ellipticals support this same picture: closed box models with $0.01Z_\odot \leq Z \leq 6Z_\odot$ have $H\beta \simeq 1.7$, while galaxies with $H\beta$ indices weaker than that are observed. Besides, the models which account for the weakest metallic indices, with $0.01Z_\odot \leq Z \leq 0.5Z_\odot$, are characterized by $H\beta \simeq 2$, while galaxies with $H\beta$ as large as 2.5 are observed. Since $H\beta$ is highly sensitive to age, one can interpret the data as an evidence for a younger age of the low metallicity Ellipticals (Gonzalez 1993). Notice however that the high Z objects do require an old age. I will further discuss this point later.

3.2 Indices versus metallicity using W models

Figure 3 displays the models for composite stellar populations obtained using Worthey's set of SSP models, with an age of 12 Gyr. The legend is the same as in Figure 1. The highest metallicity considered in the Z distributions is now $\simeq 3Z_\odot$. The qualitative effects of the presence of a metallicity spread in the stellar population are the same as already discussed for B models, and the indications derived from the comparison with the observations are very similar. Also the quantitative effects are close to those derived using for B models: the model with $0.01Z_\odot \leq Z \leq 3Z_\odot$ has $(Mg_2, Fe52, Fe53, H\beta) \simeq (0.23, 2.8, 2.5, 1.7)$ for $y = 2Z_\odot$, its average metallicity is $[<Fe/H>] = 0.11$, while a pure SSP with the same Mg_2 has $[Fe/H] = -0.16$. Compared to Buzzoni's, W SSP models have a steeper dependence of Mg_2 on $[Fe/H]$ (see also WFG), higher $Fe53$ and lower $H\beta$ over all the metallicity range. On the other hand for increasing $[Fe/H]$, the bolometric corrections to the V and B bands decrease more rapidly in Worthey's models, leading to a relatively lower contribution of the high Z SSPs in equations (10) to (13). The two effects conspire to yield the same quantitative results on the integrated indices in the CSPs.

4 COMPARISON OF CSP MODELS WITH THE OBSERVATIONS

I will now compare more in detail the predictions of the CSP models illustrated in the previous section to the observations of the line strengths in the nuclei of Ellipticals. I concentrate on some particular aspects which can be crucial for understanding the modalities of galaxy formation.

4.1 The strong Mg_2 of Giant Ellipticals

As shown in the previous subsection, a closed box model for the chemical enrichment in which the gas is initially at $Z \simeq 0$ fails to produce a stellar population with Mg_2 as high as observed in the nuclei of the brightest Ellipticals. One way to solve the problem is assuming that the initial metallicity in the gas is larger than 0. To investigate this in more detail, I have computed a sequence of models characterized by a fixed $Z_m = 3Z_\odot$ and Z_m decreasing from Z_m to $0.05Z_\odot$. The results are shown in Figure 4 for W 17 Gyr and B 15 Gyr models, where the histogram shows the Mg_2 distribution of the Ellipticals in the Davies et al. (1987) sample, which

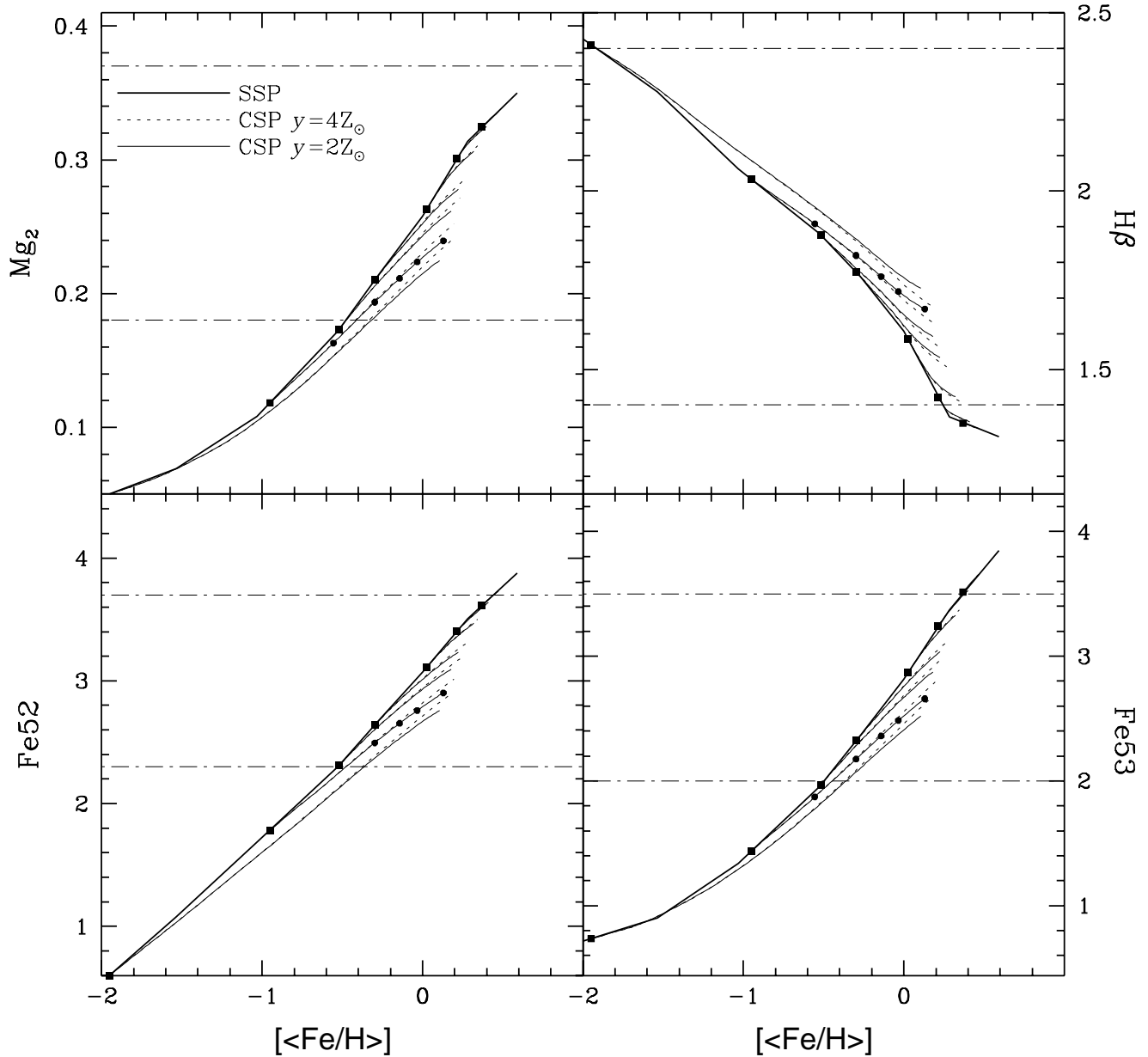


Figure 3. The same as Figure 1, but for Worthey's 12 Gyr old SSPs.

includes 469 objects, $\sim 93\%$ of which have Mg_2 in excess of 0.22.

It appears that galaxies with Mg_2 higher than 0.3 (more than 40 % of the total sample) require Z_m larger than $0.5Z_\odot$ for both W and B models, in their nuclei. Integrated Mg_2 indices in excess of 0.32 are obtained with Z_m larger than $\sim Z_\odot$, for W, or than $\sim 1.5Z_\odot$ for B SSPs, and only the models computed with Worthey's SSPs account for the largest Mg_2 . Notice that galaxies with nuclear Mg_2 as large as 0.4 have been observed, and that the adoption of a higher value for the yield (dotted lines) does not solve the problem.

The difference in the results obtained using the two sets of models is due to the different ages of the SSPs and to the different values of Mg_2 at the various metallicities. Had I

used Buzzoni's SSPs with Z in excess of $3Z_\odot$, integrated Mg_2 larger than 0.32 would have required $Z_\odot \leq Z \leq 6Z_\odot$. On the other hand, using Worthey's 12 Gyr SSPs, the integrated Mg_2 for a metallicity distribution with $2Z_\odot \leq Z \leq 3Z_\odot$ is $\simeq 0.33$. Therefore, these two sets of SSPs require that **the nuclei of the strongest Mg_2 ellipticals host both old and high metallicity SSPs, with a narrow metallicity dispersion**. The older the population, the larger the allowed metallicity range, but the presence of a significant component of stars with $Z < Z_\odot$ in galaxies with Mg_2 in excess than ~ 0.32 seems unlikely, due to the strong effect it would have on the integrated index.

Weiss et al. (1995) derive metallic line strengths for SSP models with total metallicity larger than solar, and

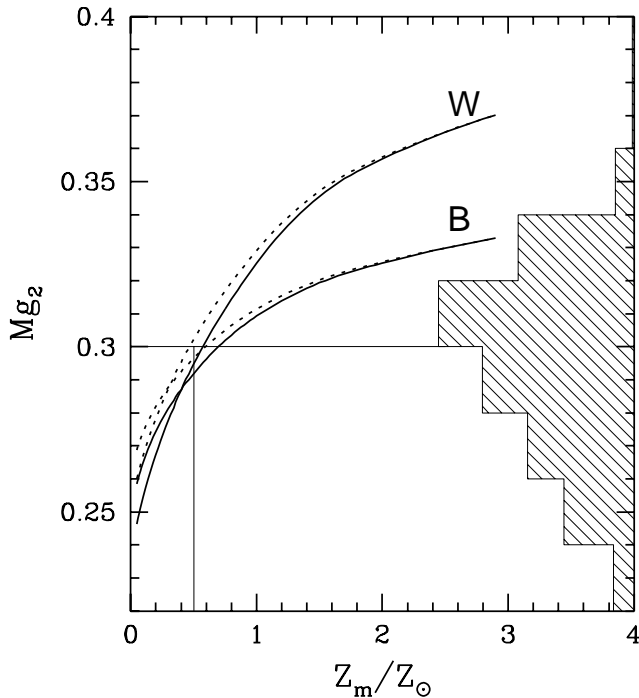


Figure 4. Dependence of Mg_2 on the parameter Z_m : the curves show the loci of CSP models characterized by a maximum value of the metallicity $Z_M = 3Z_\odot$, while Z_m is made to decrease. Solid and dotted lines correspond to $y/Z_\odot = 2$ and 3 respectively. The curves labelled W have been computed with Worthey's 17 Gyr old models; those labelled B with Buzzoni's 15 Gyr old ones. Also plotted is the distribution of the Mg_2 observed values for early type galaxies from Davies et al. (1987). Objects with Mg_2 in excess of 0.3 require Z_m greater than $\simeq 0.5Z_\odot$ for both sets of SSP models.

different elemental ratios. For solar abundance ratios, their Mg_2 indices happen to be in close agreement with those of B and W SSPs at Z_\odot , but are instead systematically larger at supersolar metallicities. For example their 15 Gyr SSP with $Z = 0.04$ has $Mg_2 = 0.4$, ~ 0.07 dex higher than B 15 Gyr model. Such high values would relieve the problem of reproducing the data for the most luminous ellipticals. However, it is likely that a closed box metallicity distribution with $Z_m \simeq 0$ would still predict too low Mg_2 values, due to the higher contribution of the low Z component to the optical light. For example, enhancing by 0.1 dex the Mg_2 indices in W 17 Gyr old SSPs at $Z > Z_\odot$, I obtain $Mg_2 = 0.27$ for a 17 Gyr old CSP with $0.01Z_\odot \leq Z \leq 3Z_\odot$. The analogous experiment with B 15 Gyr models yields $Mg_2 = 0.30$. Therefore, the need for a small metallicity dispersion in the nuclei of the strongest Mg_2 Ellipticals is a robust result.

4.2 Magnesium to Iron overabundance

The plot of the iron line strengths as functions of Mg_2 allows to check the internal consistency of the indications derived from the different indices on the composition of the stellar population. In Figure 5, a selection of the models described in the previous section (large open symbols, see captions) is

compared to the data from WFG and Gonzalez (1993) (small symbols), relative to the nuclear values of the indices. The solid lines show the locus described in this diagram from the sequence of CSP models with $Z_m = 0.01Z_\odot$ and Z_M increasing up to $3Z_\odot$. It can be seen that, in these diagrams SSPs add as vectors: the effect of a Z distribution is that of shifting the model along the SSP line (WFG), to an intermediate position between those corresponding to SSP models with $Z = Z_m$ and $Z = Z_M$. Besides, the line strengths of SSP models are stronger than those of CSP models with the same (average) metallicity: this can be seen in Figure 5 comparing the position of the skeletal symbols with that of the corresponding polygons. Once again, this reflects the larger weight of the low metallicity component on the CSP line strengths.

It appears that the average location of galaxies with $Mg_2 \leq 0.3$ is well reproduced by the models, better with Worthey's than with Buzzoni's SSPs. However, CSP models with $Z_m = 0.01Z_\odot$ barely reach $Mg_2 \sim 0.26$, $Fe52 \sim 2.9$, $Fe53 \sim 2.7$, encompassing the range occupied by the weakest Mg_2 objects. Galaxies with stronger metallic nuclear indices require some degree of preenrichment in their centers, within this class of CSP models. Notice that this is needed to account for both Mg_2 and Iron line strengths.

Those objects characterized by Mg_2 larger than approximately 0.3 (mostly represented in Gonzalez's sample) depart from the general ($Fe52, Fe53$) – Mg_2 relation, exhibiting lower Iron indices than the model predictions, at the same magnesium index. This has been interpreted as evidence for a Mg/Fe abundance ratio larger than solar in these (most luminous) ellipticals. According to the current view of how the chemical enrichment proceeds in galaxies, Magnesium is mostly produced by massive stars exploding as Type II SNe, while a substantial fraction of the Iron is provided by Type Ia SNe. Thus, a Mg to Fe overabundance can be obtained either by increasing the relative number of Type II to Type Ia events (e.g. with a flatter IMF), or by stopping the SF process at early times, i.e. before a substantial amount of pollution by Type Ia events has taken place (WFG, Davies et al. 1993, Matteucci 1994). Both scenarios predict that all the α elements, mainly produced in massive stars, are overabundant with respect to Iron. It is then reasonable to assume that the [Mg/Fe] enhancement actually traces and α elements overabundance (or an Fe underabundance), with respect to the solar ratios. In the following section I will derive a simple rule which enables to estimate the effect of an α elements overabundance on the Mg_2 and Iron line strengths.

4.3 The effect of α element enhancement on the metallic line strengths

The metallic line strength of SSPs are sensitive to the metallicity through two effects: one connected to the change of the overall shape of the isochrone, and the other to the dependence on the metal abundance of the specific feature in the individual stars which populate the isochrone, described by the fitting functions. Both effects are such that enhancing the metal abundance, the metallic features get stronger, but they operate in different ways. I try now to estimate how the shape of the isochrone on one side, and the fitting functions

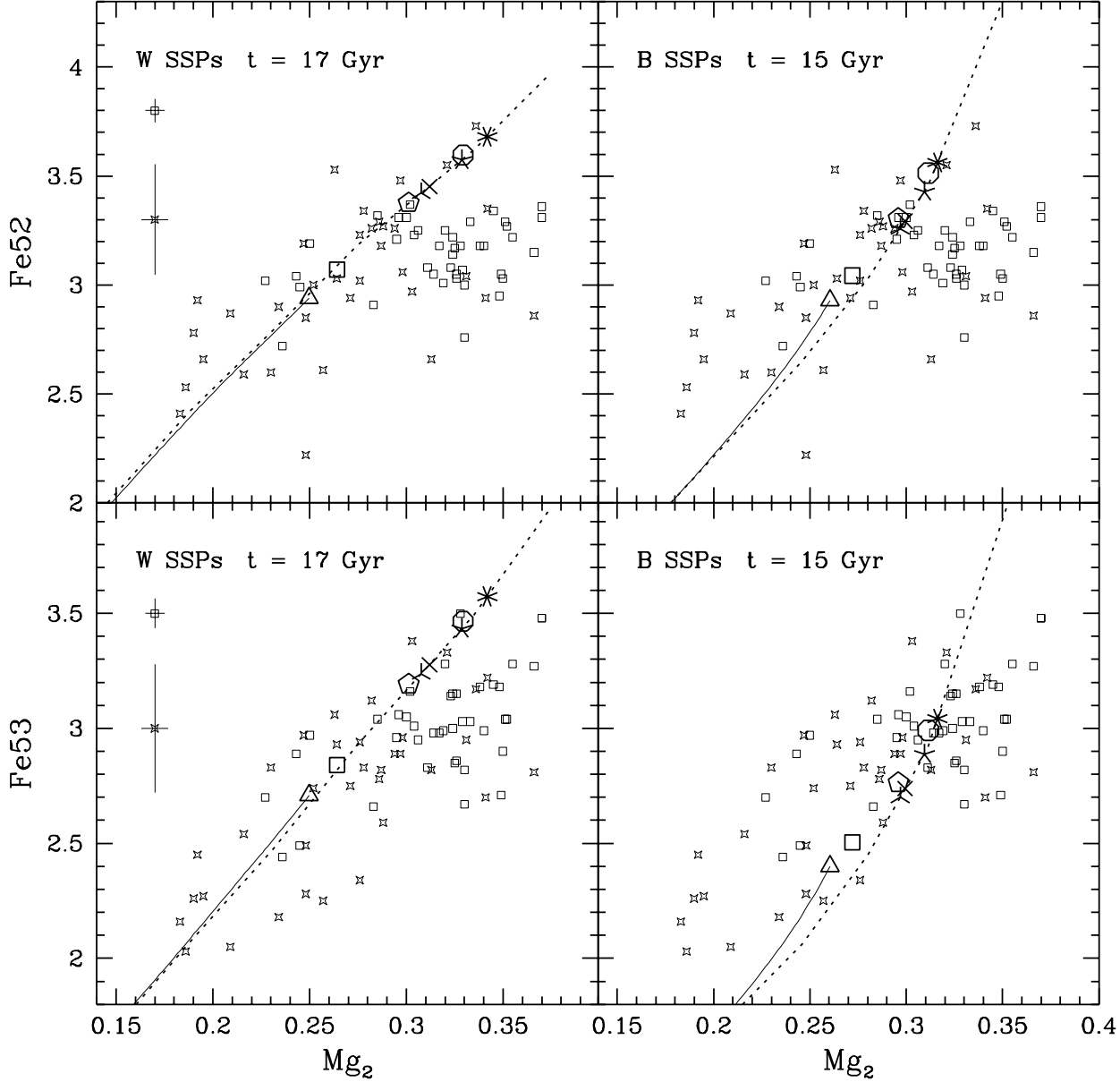


Figure 5. Comparison between model predictions and observations of the nuclear line strengths. The data points (shown as small symbols) are from Worthey, Faber & Gonzalez, 1992 (crosses) and Gonzalez, 1993 (squares). The error bars quoted by the authors are shown in the two left panels, the smallest being relative to Gonzalez data. SSP models by Worthey and Buzzoni are displayed as dotted lines. The large symbols show the line strengths for a selection of CSP models: $Z_m=0.01Z_\odot$ (triangle), $Z_m=0.1Z_\odot$ (square), $Z_m=0.5Z_\odot$ (pentagon) and $Z_m=Z_\odot$ (octagon), all having $Z_M = 3Z_\odot$ and $y = 3Z_\odot$. The sequence of CSP models obtained with $Z_m = 0.01Z_\odot$ and increasing Z_M is shown as a solid line. Notice that the closed box simple model fails to account for the high Mg_2 and Iron line strengths shown by most of the data points. The skeletal symbols show the indices of SSPs having $[Fe/H]$ equal to the average metallicity of those CSP models shown as polygons with the same number of vertices. For example, the cross shows the SSP line strengths for a metallicity equal to the average $[Fe/H]$ of the CSP shown as a square.

on the other, vary in response to an α elements overabundance.

The Mg_2 and Fe52 line strengths for SSPs are given by:

$$(Mg_2)^S = -2.5 \text{ Log } \frac{\int_{iso} n(x) f_c(x) 10^{-0.4 Mg_2(x)} dx}{\int_{iso} n(x) f_c(x) dx} \quad (15)$$

$$(Fe52)^S = \frac{\int_{iso} n(x) f_c(x) Fe52(x) dx}{\int_{iso} n(x) f_c(x) dx} \quad (16)$$

where x describes the (Log g , Log T_e) values along the isochrone, $n(x)$ and $f_c(x)$ are the number of stars and the continuum flux in the relevant wavelength bands, and

$Mg_2(x)$, $Fe52(x)$ are the fitting functions, all quantities being computed at the point x .

For a given age, $n(x)$, $f_c(x)$ and the fitting functions depend on the total metallicity Z and on the fractions $\zeta_i = X_i/Z$ of all the different elements (X_i being the abundance by mass of the i -th element). The effect of changing the fractions ζ_i (at fixed total metallicity Z) from solar ratios to α enhanced ratios on the shape of the isochrone is likely to be small. This has been shown to be valid (at low metallicities) for the turn-off region by Chaboyer, Sarajedini & Demarque (1992) and Salaris, Chieffi & Straniero (1993). Considering the RGB, its location essentially depends on the abundance of heavy elements with low ionization potentials (Renzini 1977), which, being the main electron donors, control the H^- dominated opacity. The RGB temperature is then controlled by the total abundance of these elements, namely Mg, Si, S, Ca and Fe (Salaris et al. 1993). The α enhanced mixtures predicted by detailed models for the chemical evolution of galaxies (Matteucci 1992) are indeed characterized by a similar value of the sum of the ζ_i fractions of Mg, Si, S, Ca and Fe. For example, it varies from 0.2, at solar ratios, to 0.18 for an α enhancement of ~ 0.4 dex. This means that a mixture with $Z = 0.02$ and solar abundance ratios has the same abundance (by mass) of electron donors as a mixture with $Z = 0.022$ and an average enhancement of all the α elements of 0.4 dex. It is then reasonable to assume that the shape of the isochrone is mainly controlled by the total metallicity, and that variations of the ζ_i fractions have only a marginal effect, provided that all the α elements are enhanced. Actually, compared to the solar ratios mixtures, the α enhanced ones in Matteucci (1992) models have a slightly lower fraction of electron donors, implying slightly warmer RGBs and, keeping fixed all the rest, lower metallic line strengths. The SSP models computed for solar ratios and α enhanced mixtures by Weiss et al. (1995) indeed support this conclusion (compare their model 7 to 7H).

I now turn to consider the metallicity dependence of the fitting functions. Buzzoni's $Mg_2(x)$ and $Fe52(x)$ are expressed as the sum of two terms, one depending only on metallicity and the other on gravity and temperature. For such form, the Mg_2 and $Fe52$ indices for SSPs can be written as:

$$(Mg_2)^S = \Theta_{Mg}([Fe/H]) + G_{iso} \quad (17)$$

$$(Fe52)^S = \Theta_{Fe}([Fe/H]) + G'_{iso} \quad (18)$$

in which Θ_{Mg} and Θ_{Fe} are exactly the dependences on $[Fe/H]$ of the $Mg_2(x)$ and $Fe52(x)$ fitting functions, while the G functions depend on the shape of the isochrone, and on how the stars distribute along it. For B models, $\Theta_{Fe} = 1.15[Fe/H]$ and $\Theta_{Mg} = 0.05[Fe/H] \simeq 0.05[Mg/H]$, since Buzzoni's fitting functions have been constructed using a sample of stars with likely solar abundance ratios. Assuming that the G functions only depend on the total metallicity Z , and that the fitting functions only depend on the abundance of the element contributing to the feature, for B models I can write:

$$(Mg_2)^S = 0.05 \text{Log} \zeta_{Mg} + h(Z) \quad (19)$$

$$(Fe52)^S = 1.15 \text{Log} \zeta_{Fe} + h'(Z) \quad (20)$$

were all the quantities dependent on total metallicity are described by h , h' , and ζ_{Mg} , ζ_{Fe} are the contribution to the total Z from Magnesium and Iron, respectively. It is now

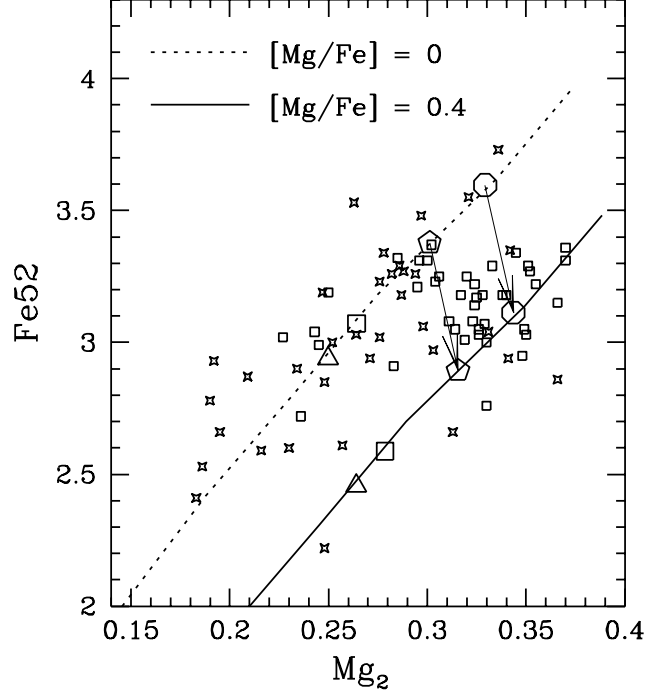


Figure 6. The effect of an α elements enhancement on the Mg_2 and $Fe52$ line strengths of W models with an age of 17 Gyr. The symbols are the same as in Fig. 5. The arrows show how the indices change for a progressively higher $[Mg/Fe]$, according to the simple scaling described in the text. It appears that the highest Mg_2 galaxies require both a larger $[Mg/Fe]$ ratio and a larger total metallicity.

easy to derive the difference between the indices for two SSPs with the same total metallicity (and age), but different α enhancements:

$$\Delta(Mg_2) = a \times \text{Log} \zeta_{Mg}^{(1)} / \zeta_{Mg}^{(2)} \quad (21)$$

$$\Delta(Fe52) = b \times \text{Log} \zeta_{Fe}^{(1)} / \zeta_{Fe}^{(2)} \quad (22)$$

with $a = 0.05$ and $b = 1.15$ for B models.

Gorgas et al. (1993) fitting functions, used in Worthey's models, do not allow the separation of the metallicity dependence from that on gravity and effective temperature. Thus, equations (17) and (18) are not strictly applicable. However, for bright RGB stars which dominate the Mg_2 index, Gorgas et al. fitting functions scale according to $\Theta_{Mg} \sim 0.19[Fe/H]$ (WFG). Most of the contribution to the $Fe52$ index comes instead from lower luminosity RGB stars (Buzzoni 1995a), for which $\Theta_{Fe} \sim 1.5[Fe/H]$ seems a fair approximation. Therefore eq. (21) and (22) can be used to estimate the effect of an α overabundance on W SSP models, with $a = 0.19$ and $b = 1.5$, since solar elemental ratios are likely to characterize also the stars in the Gorgas et al. sample.

To summarize, the line strengths of SSPs with α elements enhancement can be scaled from those of SSPs with the same age and total metallicity Z (but solar elemental ratios) using relations (21), (22) provided that:

i) the shape of the isochrone (at given age and Z), and the distribution of stars along it, are the same;

- ii) the dependence of the fitting functions on the metallicity is linear in $[M/H]$ and can be separated from the other dependences;
- iii) a given index depends only on the abundance of the element which gives rise to the considered feature.

These three requirements are never strictly true; nevertheless within the current understanding they seem to be valid approximations. Chemical evolution models by Matteucci (1992) with an α elements enhanced mixture with $[O/Fe] = 0.45$ are characterized by $\zeta_{Mg}/\zeta_{Mg,\odot} = 1.19$ and $\zeta_{Fe}/\zeta_{Fe,\odot} = 0.48$. It follows that, for this kind of α enhancement, one expects that at each metallicity point Mg_2 increases by ~ 0.004 and $Fe52$ decreases by ~ 0.37 for B models. For W models the expected differences are larger: Mg_2 increases by ~ 0.015 and $Fe52$ decreases by ~ 0.48 . The results of the application of these scaling relations are shown in Figure 6, for W 17 Gyr old SSPs. One can see that, as long as $[Mg/Fe]$ is fixed, the theoretical indices describe a locus parallel to the solar ratio sequence. Therefore, the flattening of the $Fe52$ vs Mg_2 relation at the high metallicity end can be obtained only assuming different $[Mg/Fe]$ ratios in the different galaxies. The average (flat) slope of the data points in the $Mg_2 > 0.3$ domain requires an $[Mg/Fe]$ increasing for increasing Mg_2 , up to an overabundance of ~ 0.4 dex. It is worth noticing that, in this simple scheme, Mg_2 still remains an indicator of total metallicity: changing the $[Mg/Fe]$ ratio at constant Z (i.e. enhancing X_{Mg} and decreasing X_{Fe} accordingly), does not lead to a substantial increase of the Mg_2 index, while the effect on the $Fe52$ index is more dramatic. As a result, the highest Mg_2 objects are still accounted for by the highest total Z populations. Finally, I notice that, in spite of leading to higher Mg_2 indices for a given Z , an $[Mg/Fe] = 0.4$ is not sufficient to account for the strongest Mg_2 values observed, unless the nuclei of these galaxies are inhabited by virtually pure SSPs of large total metallicity. Supersolar CSPs barely reach $Mg_2 = 0.34$, and metallicity distributions with $Z_m = 0.5Z_\odot$ don't go beyond $Mg_2 = 0.32$. Therefore, the need for a small metallicity dispersion in the nuclei of the most luminous ellipticals still remains, even assuming an overabundance which accounts for their relatively low $Fe52$ indices.

4.4 The $H\beta$ index

The $H\beta$ line strength is very sensitive to the temperature of turn-off stars; thus plotting $H\beta$ versus an index which is mainly controlled by Z may allow to estimate independently age and metallicity of a stellar system (see e.g. Gonzalez 1993). Adopting this approach, Faber et al. (1995) suggest that the nuclei of elliptical galaxies form more an age sequence of high Z objects, as opposed to a metallicity sequence of old objects. In their comparison, the effect of a non zero $[Mg/Fe]$ is taken into account plotting $H\beta$ versus a newly defined index ($[MgFe]$), equal to the geometric mean of Mgb and $\langle Fe \rangle$. This index is meant to trace better the total metallicity Z , but there is no guarantee that it actually does (see also Faber et al. 1995). I prefer to use Mg_2 as metallicity indicator, and account for the Magnesium overabundance with the simple scaling given by relation (21).

Figure 7 shows the locus described by SSP and CSP models in the $H\beta$ vs Mg_2 plane for various ages, together

with Gonzalez (1993) data. In order to mimic the effect of an α elements enhancement, a constant shift has been applied to the SSPs Mg_2 values, which amounts to 0.015 dex for W and 0.004 dex for B models, corresponding to a constant overabundance of $[Mg/Fe] = 0.4$.

The CSP models in Figure 7 appear to support Faber et al. (1995) conclusion, and further show that a metallicity spread would substantially worsen the agreement between the models and the observations, at all metallicities. Actually, the dot dashed line in the left panel of Figure 7, fairly fitting the observations, connects the $Z = 3Z_\odot$ SSPs in the W set of models. Since in this diagram SSPs add as vectors, an internal age spread of ~ 1 -2 Gyr in the CSPs would hardly affect the interpretation of the data, which seem to require younger average ages for the lower metallicity objects. However, since the low Mg_2 galaxies are also the fainter Ellipticals in the local sample, a tight mass–age relation would be implied, with the less massive Ellipticals being (on the average) younger than the most massive ones.

Basically the same conclusion holds when considering B models (right panel): in spite of predicting $H\beta$ line strengths which are systematically higher than Worthey's, at any age and Mg_2 , still the 15 Gyr old locus is too shallow with respect to the data. In essence, for old ages the model $H\beta$ index is too low at low metallicities, and ages younger than 8 Gyr are required to fit the $H\beta$ values of the low Mg_2 galaxies.

The need for invoking an age difference in the galaxies of Gonzalez (1993) sample is related to the mild dependence of the $H\beta$ line strength on the metallicity, which reflects the dependence on Z of the turn–off temperature. This is the case for SSP models with Red Horizontal Branches (RHB). On the other hand, as is well known, the HB stars in the galactic globular clusters become bluer with decreasing metallicity, although the trend is not strictly monothonic due to the *second parameter* problem (see e.g. Renzini 1977). If the average temperature of HB stars increases for decreasing metallicity, $H\beta$ will be more sensitive to Z than estimated in the SSP models considered until now. As a result, the presence of a low metallicity tail in the stellar populations in Ellipticals could affect the CSP $H\beta$ line strengths appreciably, leading to higher equivalent widths the larger the population in the low Z tail. According to Buzzoni et al. (1994), an Intermediate Horizontal Branch (IHB) (corresponding to a temperature distribution peaked at $\log T_e = 3.82$, and a blue tail extending up to $\log T_e = 4.05$, such as e.g. in the globular cluster M3) leads to $H\beta$ indices higher by $\simeq 0.7 \text{ \AA}$, with respect to SSPs with RHB. Thus, in order to estimate the impact of this effect on the $H\beta$ line strength I have computed integrated indices for CSPs using Buzzoni's 15 Gyr old models with an artificially increased $H\beta$. The enhancement is taken equal to 0.5 \AA for all metallicities less than $\sim 0.6Z_\odot$, and linearly vanishing at Z_\odot . The effect of the adoption of an IHB on the Mg_2 index has instead been neglected, since it is estimated to lead to a decrease of only a few 10^{-3} mag (Buzzoni et al. 1994). Notice that a $[Mg/Fe] = 0.4$, which would correspond to an increase of Mg_2 by approximately the same amount for B models, has not been taken into account in this computation.

The results are shown in Figure 8, where the locus described from CSPs with various Z_m is displayed. Obviously, those CSPs with Z_m in excess of $\sim 0.5Z_\odot$ are not affected by

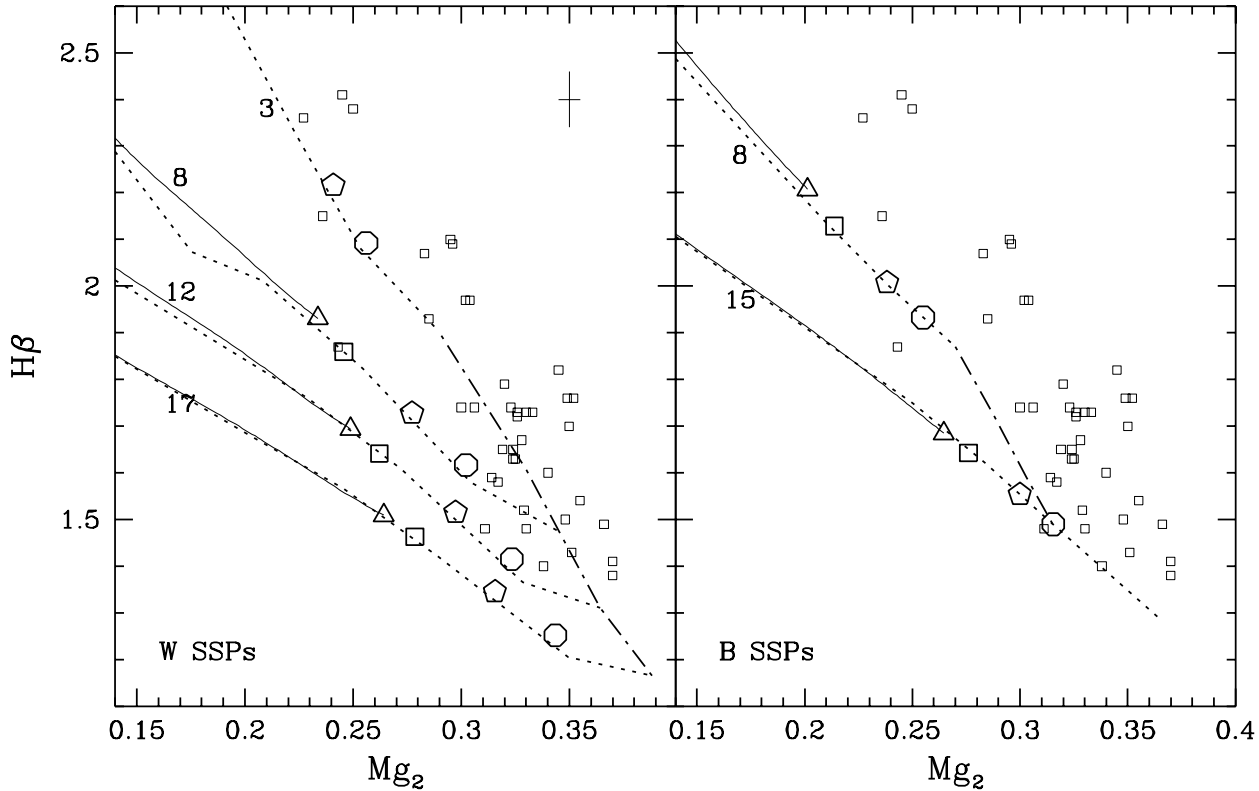


Figure 7. Comparison between observations of $H\beta$ and Mg_2 indices in the nuclei of Ellipticals and model predictions using Worthey's (left panel) and Buzzoni's (right panel) SSPs. The data points (small squares) are from Gonzalez (1993) and the error bar quoted by the author is shown in the left panel. The dotted lines connect SSP models of constant age and different metallicities, each line labelled with its age in Gyr. The dot-dashed lines connect constant metallicity SSPs with $Z = 3Z_\odot$ (left panel), $1.7Z_\odot$ (right panel). The solid lines are the loci described by single age CSPs, $y = 3Z_\odot$, $Z_m = 0.01Z_\odot$, and Z_M increasing up to $3Z_\odot$, for the various ages. The big symbols indicate the location in this diagram of CSPs with fixed $Z_M = 3Z_\odot$ and different Z_m . The encoding is the same as in Figure 5, except for the following cases: the 3 Gyr old CSP shown as a pentagon has $Z_m = 0.6Z_\odot$ (instead of $0.5Z_\odot$); the 8 Gyr old CSPs based on Buzzoni's models have been computed with an upper metallicity cut off of $Z_M \simeq 1.7Z_\odot$ (instead of $3Z_\odot$). This different limits are due to the limited Z range covered by the SSP models at these ages. A constant shift of 0.015 (for Worthey's models) and 0.004 (for Buzzoni's) has been applied to the theoretical Mg_2 line strengths to account for the $[Mg/Fe]$ overabundance.

these new prescriptions, and their corresponding line indices lay on the locus of RHB SSP models. On the contrary, CSPs with sufficiently low Z_m have substantially higher $H\beta$, for a given Mg_2 , and the data in this diagram could be interpreted as a sequence of old stellar populations, with increasing average metallicity. While the need for a small metallicity dispersion in the nuclei of the most luminous Ellipticals discussed in the previous sections leaves little room for a sizeable contribution of a low Z component, the less luminous Ellipticals can host a substantial low Z , IHB component in their nuclei. This experiment shows that the strength of the $H\beta$ line is very sensitive to the temperature distribution assumed for HB stars. Actually, just comparing Fig. 8 to the right panel of Fig. 7, it appears that the data can be equally interpreted as an age sequence, at a constant high metallicity, or as a sequence at constant old age of CSPs with decreasing metallicity spread, corresponding to an increasing average metallicity.

5 DISCUSSION

The numerical experiments performed in this paper illustrate how the Mg_2 , Fe52, Fe53 and $H\beta$ line strengths are affected by the presence of a metallicity distribution shaped like the closed box model predictions. I have explored systematically the results of changing the minimum and maximum metallicity (Z_m , Z_M) characterizing the chemical processing: the first parameter describes the possibility of pre-enrichment of the gas in the closed box to different degrees; the second, the occurrence of galactic winds, inhibiting further chemical processing at various levels of completion. I summarize now the major results, and derive some hints on the stellar population inhabiting the central regions of Ellipticals.

5.1 The average metallicity in the nuclei of Ellipticals

Due to its major contribution to the light in the optical bands, the low metallicity component tends to dominate

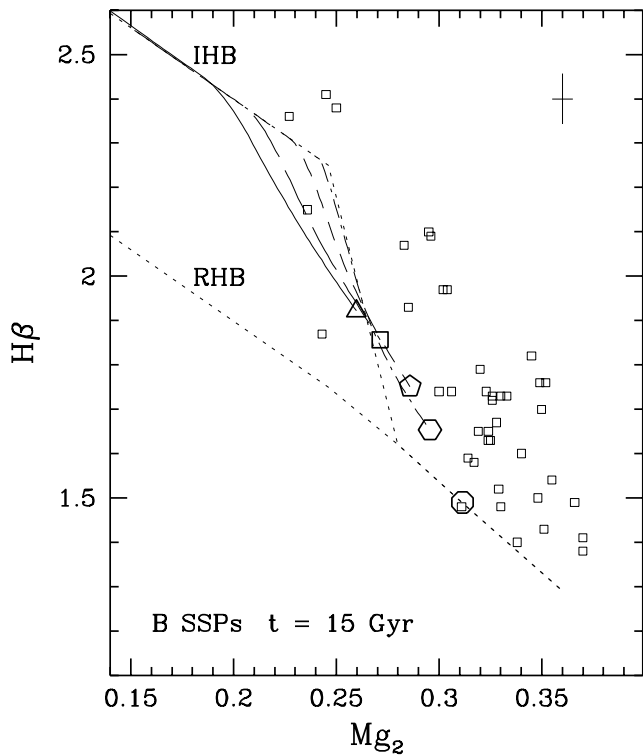


Figure 8. Effect of the contribution of an IHB on the $H\beta$ index. SSP models from Buzzoni with an age of 15 Gyr are shown as dotted lines for a RHB and an IHB (see text). CSP model sequences computed with IHB SSPs are shown as different lines, along which Z_m increases up to $3Z_\odot$. All the CSP models have $y = 3Z_\odot$, while $Z_m/Z_\odot = 0.01, 0.1, 0.3, 0.5$ for the solid, long dashed, short dashed and dot-dashed lines, respectively. The last CSP model of each sequence is shown as a big symbol. The big octagon, on the RHB line, shows the line strengths for the CSP model with $Z_m = Z_\odot$, $Z_M = 3Z_\odot$, $y = 3$. No shift to higher Mg_2 values to account for the possible $[Mg/Fe]$ enhancement has been applied here.

the integrated indices. This implies that the metallic indices of the composite stellar populations are systematically weaker than those of simple stellar populations, with the same metallicity. Therefore, a given value for a metallic line strength, corresponds to CSPs with larger mass averaged metallicities than SSPs. The difference between the two metallicities depends on the width of the Z distribution in the CSP model, and it can be as large as ~ 0.3 dex. As a consequence, a quantitative relation between metallic line strength and average metallicity is subject to a substantial uncertainty: it depends on the metallicity distribution, which cannot be constrained, and on the SSP models, that may still be affected by inadequacies. Indeed, some differences are present between the various sets of models available in the literature (see also Charlot, Worthey & Bressan 1996), which affect the relation between integrated indices and the average metallicity of a given CSP. This impacts on the calibration of the spectral indices of different galaxies in term of their metallicity, as well as on the derivation of abundance gradients from line strength gradients within a given galaxy.

5.2 The metallicity spread in the nuclei of Ellipticals

The systematic exploration of the influence of the Z_m and Z_M parameters shows that the sequence from low to high luminosity ellipticals (as far as their central stellar population is concerned) can be interpreted in various ways: as a sequence of virtually pure SSPs of increasing metallicity; as a sequence of CSPs in which either the low metallicity component becomes less and less important, or both the minimum and the maximum metallicity increase. Chemical evolution models with galactic winds rather predict a metallicity sequence among Ellipticals characterized by an increasing Z_M for increasing mass (and luminosity) of the galaxy, with an important low metallicity component always present. This class of models can account for objects with Mg_2 , Fe52 and Fe53 up to ~ 0.27 , 3 and 2.7 respectively, while galaxies with higher values of these indices cannot be reproduced. Notice that $\sim 80\%$ of objects in the Davies et al. (1987) sample has $Mg_2 \geq 0.26$. Since these considerations apply only to the nuclear indices, classical wind models can still be adequate to describe the global metallicity distribution in Ellipticals, but a mechanism should be found to segregate the high metallicity component in the nuclei.

A similar problem has been found by Bressan, Chiosi & Fagotto (1994), when comparing the spectral energy distribution of their model Ellipticals with the observations. These authors pointed out that the theoretical metallicity distribution was too heavily populated in the low metallicity range, leading to an excess light between 2000 and 4000 Å with respect to the observed spectra. They noticed that a much better fit was achieved if a minimum metallicity of $Z = 0.008$ was assumed, and concluded that the classical chemical evolution models for elliptical galaxies, like those for the solar neighborhood, are affected by the *G-dwarf problem*, i.e. the excess of low metallicity stars predicted by the closed box model for the solar neighborhood. The classical solutions to *cure* the G-dwarf problem are (see Audouze & Tinsley 1976):

- i) infall of metal free gas, so that the SFR exhibits a maximum at some intermediate epoch, when a substantial enrichment has been already accomplished;
- ii) prompt initial enrichment (PIE), like what explored here by varying the Z_m parameter;
- iii) adopting a SFR enhanced in high metallicity gas, in which the stars are formed with a larger metallicity than the average Z of the interstellar medium.

A variation of the PIE model consists in assuming that the first stellar generations are formed with a conveniently flat IMF (Vazdekis et al. 1996), so that they contribute metals at early times, but not light at the present epoch. The following generations would instead form with a normal IMF.

All these are in principle viable solutions, and which, if any, of these applies to the nuclei of Ellipticals remains to be seen. However, I notice that the infall models predict for the most massive galaxies Mg_2 indices not larger than 0.28 (Bressan et al. 1996): still a low value with respect to the observations in the nuclei of massive Ellipticals. Solution iii) requires large inhomogeneities in the gas, and a variable IMF seems a rather *ad hoc* solution.

A prompt initial enrichment for the gas in the nuclei of Ellipticals is easily realized by relaxing the hypothesis of

instantaneous complete mixing of the gas, and allowing enriched gas to sink to the center. This is indeed a natural result of dissipational galaxy formation (Larson 1975). During its formation process, a galaxy consists of two components: one dissipationless (the newly formed stars), and one dissipative (the gas). Once formed, the stars stop participating to the general collapse, keeping thereafter memory of their energy state at formation. The gas, instead, will continue to flow towards the center, being progressively enriched as SN explosions take place. Thus, gas accumulated in the nuclear regions is pre-enriched by stars which formed (and died) in the outer regions, and will further form stars, which will then be the most metal rich in the galaxy. The metal poor stars missing in the galaxies nuclei should be found in the outer regions. In other words, the different dissipation properties of the stars and the gas would lead to a chemical separation within the galaxy, during its formation, no matter whether the protogalaxy is a monolithically collapsing cloud, or if it consists of gas rich merging lumps. Interestingly, this out-inward formation, would also allow for peculiarities in the core kinematics, as observed in a substantial fraction of galaxies (Bender et al. 1993), since the formation of the nucleus would be partially decoupled from the formation of the rest of the galaxy.

5.3 The elemental ratios in the nuclei of Ellipticals

The Iron indices vs Mg_2 plot suggests the presence of a Magnesium overabundance in the brightest Ellipticals ($Mg_2 > 0.3$), as pointed out by WFG comparing the data to SSP models. The presence of a metallicity distribution does not alter this conclusion, since the CSP models considered here describe the same locus of SSPs in the Iron vs Mg_2 diagram. If the Magnesium overabundance is tracing a true α elements overabundance, it is possible to estimate the enhancement quantitatively using a simple scaling of the SSP models constructed for solar abundance ratios. This estimate however is very sensitive to the dependence of the Mg_2 index on the Mg abundance, and of the Iron index on the Fe abundance. Assuming that these dependences are the same as the metallicity dependence of the relative fitting functions, for Worthey's models I have found that the brightest ellipticals should be characterized by $[Mg/Fe]$ ratios ranging from 0 to 0.4 approximately, for increasing Mg_2 . It may seem that a Magnesium overabundance would easily explain the strong Mg_2 values without invoking high total metallicities in the nuclei of the brightest Ellipticals. However, $[Mg/Fe] > 0$ means Mg enhancement together with Fe depletion, with respect to the solar ratio. In the frame of an overall α element overabundance, the chemical evolution models actually predict a lower fraction of electron donors at fixed total metallicity, as mentioned in Section 3.4. Correspondingly, the temperature of the RGB is increased, counterbalancing the increase of the Mg_2 index conveyed by a higher Mg abundance. Thus, large metallicities and small metallicity dispersions are still needed to account for the data.

According to current modelling an overabundance of $[Mg/Fe] = 0.4$ implies very short formation timescales for the whole galaxy, since the Mg and Fe gradients, within the errors, are the same (Fisher, Franx & Illingworth 1995).

How short is interesting to look at: Matteucci (1994) models for the chemical evolution of Ellipticals predict that a solar $[Mg/Fe]$ is reached already at 0.3 Gyr. An overabundance implies formation timescales shorter than this, and the higher $[Mg/Fe]$, the shorter the timescale required. Indeed, in Matteucci (1994) *inverse wind models*, the galactic wind occurs at only ~ 0.15 Gyr in a $5 \times 10^{12} M_\odot$ galaxy, and yet the corresponding overabundance is not larger than 0.3. The formation timescales inferred from a given $[Mg/Fe]$ overabundance depend on the adopted model for the Type Ia supernovae progenitors, and accordingly can be considered quite uncertain. It seems however unlikely that a Mg to Fe overabundance could be accomplished with formation timescales longer than ~ 1 Gyr. For example, Greggio (1996) finds that, following a burst of Star Formation, $\simeq 50\%$ of the total Iron from the Type Ia SNe is released within a timescale ranging from 0.3 to 0.8 Gyr, for a variety of SNIa possible progenitors.

5.4 The $H\beta$ line strength as age indicator

As for the Iron indices, the comparison of models to observations in the $H\beta$ vs Mg_2 plot does not change if a metallicity spread in the stellar populations is taken into account. In SSPs with red horizontal branches the only way to enhance $H\beta$ is by adopting a warmer turn-off, that is younger ages. In this case, the constraint from the metallic indices on the metallicity and metallicity dispersion discussed until now is even stronger, since younger ages make Mg_2 , Fe52 and Fe53 weaker. Extremely large metallicities should characterize the nuclei of all Ellipticals in Gonzalez (1993) sample, with virtually no metallicity dispersion. If however the temperature distribution of HB stars is wide enough, strong $H\beta$ indices can be obtained in old stellar populations, due to the contribution of *warm* HB stars. In this case, the $H\beta$ vs Mg_2 plot would trace the different proportions of this stellar component. On the average, the effect is stronger for the less metallic galaxies, if they host a composite stellar population with larger fraction of low Z stars, and the data are consistent with an old age for the stars in the centers of this sample of Ellipticals.

The nuclei of galaxies with Mg_2 in excess of ~ 0.3 are however likely to host only stars with metallicity larger than $\sim 0.5Z_\odot$. For these galaxies, there is little room for a warm HB component, at least in the frame of the canonical stellar evolution, and yet their $H\beta$ line strengths range from ~ 1.4 to ~ 1.8 Å. Adopting Worthey's $3Z_\odot$ SSP models, this implies and age range from ~ 12 to ~ 5 Gyr, if the stellar populations in the nuclei of these galaxies are coeval. If, however, I consider a two component population, one very old and one very young, relatively high $H\beta$ values can be accomplished with a small contribution to the total light (and even smaller to the total mass) from the young component. For example, a combination of a 17 Gyr plus a 1.5 Gyr old SSP, both with 3 times solar metallicity, contributing 80 % and 20 % of the light respectively, has $H\beta \simeq 1.74$ and $Mg_2 \simeq 0.32$. The contribution to the total mass of the young SSP would amount to only ~ 7 %. Adopting a solar metallicity for the young component, one gets $H\beta \simeq 1.8$, $Mg_2 \simeq 0.32$ with only 10% of the light coming from the 1.5 Gyr old population, corresponding to a $\sim 3\%$ contribution

to the total mass. Indeed, owing to the steep age dependence of $H\beta$, a small fraction of light from the young stellar population is sufficient to enhance the $H\beta$ line strength. If this was the case, the bulk of the population in the nuclei of these galaxies would be truly old, with $H\beta$ tracing a relatively unobscured recent Star Formation event.

6 CONCLUSIONS

Using the SSP models currently available in the literature to construct integrated indices for composite stellar populations with a metallicity spread I have shown that the nuclei of the most luminous elliptical galaxies should host stellar populations with:

- a) high total metallicity;
- b) a Magnesium overabundance with respect to Iron, with varying degrees of the $[Mg/Fe]$ ratio.
- c) a small metallicity spread.

Condition a) is met by processing the gas through multiple stellar generations, and condition b) requires that this processing occurs within a short time scale. This inevitably means that during the chemical enrichment the star formation rate was very high, implying a correspondingly large SNII rate. In order to proceed with the chemical processing, the gas had to be subject to confinement, that is it had to be located within a deep potential well. Condition c) is met if the gas turning into stars in the nuclei of Ellipticals has been substantially pre-enriched, or if the maximum SFR was achieved at some late stage, when partial chemical processing had already been completed, like in infall models. However, for any behaviour of the SFR with time, as long as one considers a selfpolluting gas mass, a low metallicity component in the final stellar population is unavoidable: it is this low metallicity component which provides the metals to build up the high Z stars. As a consequence, the extremely high Mg_2 indices would rather favour the pre-enrichment alternative.

These facts support the notion that the gas out of which the nuclei of the most luminous ellipticals formed was produced within the galaxy itself, and was not accreted from the outside. Several evidences indicate that merging must have played a role in the formation of these galaxies, including the relatively shallow metal line gradients (e.g. Davies et al. 1993), and the peculiar kinematics in the nuclei of a substantial fraction of galaxies (see Bender 1996). The indications from the analysis performed in this paper suggest that the merging subunits should have been mostly gaseous, and confined within the deep potential well in which the galaxy itself is found. The segregation of the high Z component in the inner parts could result from the gas participating of the general collapse more than the stars, which could have formed within the merging subunits.

As discussed in the previous section, the indications from the $H\beta$ line strength are sufficiently ambiguous that the possibility that the bulk of the stars in the nuclei of the brightest Ellipticals are indeed old remains favourable. Therefore, the formation of these galaxies should have occurred at high redshifts, both the stellar component and the potential wells, within which the chemical processing could proceed up to high metallicities in short time scales.

For the lower luminosity Ellipticals the conclusions are

more ambiguous, as their nuclear line strengths are consistent with both wide and narrow metallicity distributions. However, galaxies with central Mg_2 in excess of ~ 0.27 (approximately 70% in Davies et al. sample) are not accounted for by closed box models with $Z_m \sim 0$. Therefore, for the majority of Ellipticals some chemical separation should have taken place during their formation, although a substantial low metallicity component could be present in their nuclei. If this was the case, the $H\beta$ line strength would trace the proportion IHB stars produced by this component of the composite stellar population in the different galaxies, which would then be old. Finally, the Mg/Fe ratio in the lower luminosity Ellipticals is likely to be solar, suggesting longer timescales for the formation of the bulk of their stars, with respect to the brighter Ellipticals.

A final caveat concerns the reliability of the SSP models used for the interpretation of the observational data. The differences among the various sets of models, the different fitting functions, and the lack in the stellar data sets of a fair coverage of the age, metallicity and $[Mg/Fe]$ ratio cast some doubt on the use of these models to derive quantitative informations. The data seem to suggest that the stars in the nuclei of elliptical galaxies have Mg_2 indices stronger than what can be reasonably inferred using these SSP models. Actually Mg_2 as large as 0.4 are difficult to account even with the oldest and most metal rich SSPs in the Worthey's and Buzzoni's sets of models. Although this may be a problem of just a few object, this fact may suggest some inadequacies in the SSP models. If the dependence of the Mg_2 index on metallicity is currently underestimated at the high Z end, lower total metallicities, larger metallicity dispersions and lower $[Mg/Fe]$ ratios would be possibly allowed for the stellar populations in the nuclei of the brightest ellipticals. Nevertheless, the presence of a small metallicity dispersion in the nuclei of giant ellipticals and the need for old ages seem to be quite robust conclusions, due to the larger contribution to the optical light of the less metallic populations.

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REFERENCES

- Audouze J., Tinsley B.M., 1976, ARA&A, 14, 43
- Arimoto N., Yoshii Y., 1987, A&A, 173, 23
- Binney J., Tremaine S., 1987, Galactic Dynamics. Princeton Univ. Press, Princeton, NJ
- Bender R., 1996, in Bender R., Davies R.L., eds, Proc. IAU Symp. 171, New Light on Galaxy Evolution. Kluwer, Dordrecht, p. 181
- Bender R., Burstein D., Faber S.M., 1992, ApJ, 399, 462
- Bender R., Burstein D., Faber S.M., 1993, ApJ, 411, 153

- Bender R., Ziegler B., Bruzual G., 1996, ApJ, in press
- Bower R.G., Lucey J.R., Ellis R.S., 1992, MNRAS, 254, 613
- Bressan A., Chiosi C., Fagotto F., 1994, ApJS, 94, 63
- Bressan A., Chiosi C., Tantalo R., 1996, A&A, in press
- Burstein D., Faber S.M., Gaskell C.M., Krumm N., 1984, ApJ, 287, 586
- Buzzoni A., 1995a, ApJS, 98, 69
- Buzzoni A., 1995b, in Buzzoni A., Renzini A., Serrano A., eds, ASP Conf. Ser. 86, Fresh Views of Elliptical Galaxies. p. 189
- Buzzoni A., Gariboldi G., Mantegazza L., 1992, AJ, 103, 1814
- Buzzoni A., Mantegazza L., Gariboldi G., 1994, AJ 107, 513
- Carollo C.M., Danziger I.J., Buson L., 1993, MNRAS, 265, 553
- Casuso E., Vazdekis A., Peletier R.F., Beckman J.E., 1996, ApJ, 458, 533
- Chaboyer B., Sarajedini A., Demarque P., 1992, ApJ, 394, 515
- Charlot S., Worthey G., Bressan A., 1996, ApJ, 457, 625
- Davies R.L., Burstein D., Dressler A., Faber S.M., Lynden-Bell D., Terlevich R.J., Wegner G., 1987, ApJS, 64, 581
- Davies R.L., Sadler E.M., Peletier R.F., 1993, MNRAS, 262, 650
- Dickinson M., 1996, in Buzzoni A., Renzini A., Serrano A., eds, ASP Conf. Ser. 86, Fresh Views of Elliptical Galaxies. p. 283
- Faber S.M., Trager S.C., Gonzalez J.J., Worthey G., 1995, in Van Der Kruit P.C., Gilmore G., eds, Proc. IAU Symp. 164, Stellar Populations. Kluwer, Dordrecht, p. 249
- Fisher D., Franx M., Illingworth G., 1995, ApJ 448, 119
- Freedman W.L., 1989, AJ, 98, 1285
- Franx M., van Dokkum P.G., 1996, in Bender R., Davies R.L., eds, Proc. IAU Symp. 171, New Light on Galaxy Evolution. Kluwer, Dordrecht, p. 233
- Greggio L., 1996, in Kunth D. et al., eds, The Interplay between Massive Star Formation, the ISM and Galaxy Evolution. Edition Frontieres, in press
- Gonzalez J.J., 1993, PhD Thesis
- Gorgas J., Efstathiou G., Aragón Salamanca A., 1990, MNRAS, 245, 217
- Gorgas J., Faber S.M., Burstein D., Gonzalez J.J., Courteau S., Prosser C., 1993, ApJS 86, 153
- Kauffmann G., White S.D.M., Guiderdoni B., 1993, MNRAS, 264, 201
- Kauffmann G., Charlot S., White S.D.M., 1996, preprint
- Larson R.B., 1975, MNRAS, 173, 671
- Lilly S.J., Tresse L., Hammer F., Crampton D., Le Fèvre O., 1995, ApJ 455, 108
- Matteucci F., 1992, Mem.S.A.It., 63, 301
- Matteucci F., 1994, A&A, 288, 57
- Matteucci F., Tornambé A., 1987, A&A, 185, 51
- McWilliam A., Rich R.M., 1994, ApJS, 91, 749
- O'Connell R., 1986, in Chiosi C., Renzini A., eds, Spectral Evolution of Galaxies. Kluwer, Dordrecht, p. 321
- Renzini A., 1977, in Bouvier P., Maeder A., eds, Advanced Stages in Stellar Evolution. Geneva Obs., Geneva, p. 151
- Renzini A., 1986, in Chiosi C., Renzini A., eds, Spectral Evolution of Galaxies. Kluwer, Dordrecht, p. 194
- Renzini A., 1995, in Van Der Kruit P.C., Gilmore G., eds, Proc. IAU Symp. 164, Stellar Populations. Kluwer, Dordrecht, p. 325
- Renzini A., Ciotti L., 1993, ApJ, 416, L49
- Rich R.M., 1988, AJ, 95, 828
- Salaris M., Chieffi A., Straniero O., 1993, ApJ, 414, 580
- Tantalo R., Chiosi C., Bressan A., Fagotto F., 1996, A&A, in press
- Tinsley B.M., 1980, Fund. Cosmic Phys., 5, 287
- Vazdekis A., Casuso E., Peletier R.F., Beckman J.E., 1996, ApJ, in press
- Weiss A., Peletier R.F., Matteucci F., 1995, A&A, 296, 73
- Worthey G. 1994, ApJS, 95, 107
- Worthey G., Faber S.M., Gonzalez J.J., 1992, ApJ, 398, 69 (WFG)